vich number; $E = Gc_h \sum_{i=1}^{n} [T_{h^0} + \frac{1}{2} (T_{h^{i-1}} + T_{h^i})] \Delta t_i$, stored energy; $E_t = m[c_S(T_m - T_0) + h + c_L(T_{ho} - T_m)] + m_s c_s (T_{h^0} - T_0)$, total amount of energy; $\tau^* = \overline{Nu}(T_{h^0} - T_0)\pi l \lambda'_h \times t/E_t$, dimensionless time. Subscripts: 0, initial state; m, melting; L, liquid phase; S, solid phase; h, heat carrier; w, wall; s, structure.

LITERATURE CITED

1. R. G. Kemink and E. M. Sparrow, Int. J. Heat Mass Trans., 24, No. 10, 1699-1710 (1981).
2. Sparrow and Broadbent, Teplopered., 104, No. 2, 85-92 (1982).
3. M. Okada, Int. J. Heat Mass Trans., 27, No. 11, 2057-2066 (1984).
4. Kho and Viskanta, Teplopered., 106, No. 1, 9-18 (1984).
5. Kal'khori and Ramad'yani, Teplopered., 107, No. 1, 42-50 (1985).
6. Benar, Goben, and Martines, Teplopered., 107, No. 4, 49-58 (1985).
7. Go and Viskanta, Teplopered., 108, No. 1, 159-166 (1986).
8. Sparrow, Gertshed, and Mairam, Teplopered., 108, No. 3, 142-147 (1986).
9. E. M. Sparrow and C. F. Hsu, Int. J. Heat Mass Trans., 24, No. 8, 1345-1357 (1981).
10. Hsu and Sparrow, Teplopered., 103, No. 3, 231-233 (1981).
11. N. Shamsundar, Int. J. Heat Mass Trans., 25, No. 10, 1614-1616 (1982).
12. K. V. Karmastin and V. A. Kuznetsov, Energy Storage and Means of Increasing the Operating Efficiency of Power Stations and Energy Savings: Proceedings of an All-Union Scientific Conference [in Russian], Moscow, Part II, pp. 136-144 (1986).
13. W. R. Humphries and E. I. Griggs, NASA Technical Paper, No. 1074 (1977).
14. S. K. Jain, J. Chem. Eng. Data, 23, No. 2, 170-173 (1978).
15. J. Guion, J. D. Sauzade, and M. Laugt, Thermochim. Acta, 67, No. 2, 167-179 (1983).

ACCURATE SOLUTIONS OF BOUNDARY-LAYER EQUATIONS FOR MOVING PERMEABLE SURFACES

G. I. Burdé

UDC 532.526

New accurate solutions of the equations of a laminar boundary layer are obtained for steady flow induced by the motion of a continuous solid surface at constant velocity.

The flow arising in the motion of a solid surface in a quiescent liquid is of interest in connection with film and fiber production in the glass and polymer industries. The boundary layers at continuous surfaces moving at constant velocity in a viscous incompressible liquid were considered in [1-5]. In [1-4], the flow close to impermeable plane and cylindrical surfaces was studied. In the case of a plane surface, when there is a self-similar solution, the problem reduces to numerical integration of an ordinary differential equation [1, 2]. In the case of a cylindrical surface, the solution is obtained by the integral method [1, 3] and the method of expansion in power series with respect to the radial coordinate [4]. In [5], the self-similar solution was investigated numerically for a boundary layer at a permeable plane surface, through which there is suction or injection of the same liquid at a velocity decreasing in the direction of motion of the plane according to the law $v_0 = q/\sqrt{x}$.

The present work gives some accurate solutions of the equations of a laminar boundary layer of incompressible liquid, describing the flow close to permeable surfaces moving at constant velocity; these solutions exist in the presence of a definite relation between the liquid suction rate and the other parameters of the problem. The solution for the case of a cylindrical surface gives a sufficiently rare example of an accurate non-self-similar solution.

The boundary-layer equations for a flow induced by the motion of a plane surface in

---

Perm' State Pedagogical Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 58, No. 2, pp. 220-223, February, 1990. Original article submitted September 19, 1988.

 0022-0841/90/5802-0162$12.50

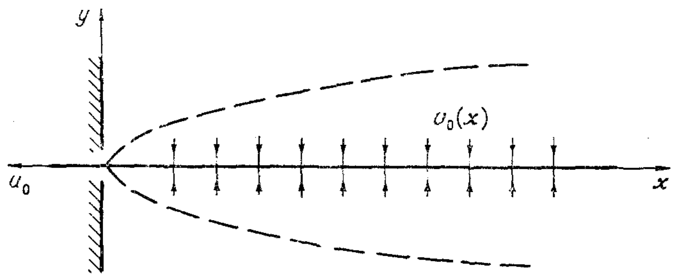

Fig. 1. Boundary layer at a moving plane surface with suction.

quiescent liquid take the form

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2},\quad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0.$$

These equations assume the solution

$$u=\frac{6u_0}{(\eta+\sqrt{6})^2},\quad v=-\frac{q}{\sqrt{x}}\frac{\sqrt{6}(2\eta+\sqrt{6})}{(\eta+\sqrt{6})^2}, \tag{1}$$

$$\eta=\sqrt{-\frac{u_0}{\nu x}}\,y,\quad q=\sqrt{-\frac{3\nu u_0}{2}},$$

which corresponds to a plate moving at velocity $|u_0|$ in the direction opposite to the x axis, in the presence of suction with an intensity distributed according to the law $v_0 = -q/\sqrt{x}$. This situation, shown schematically in Fig. 1, differs from that considered in [1, 2, 5] in the direction of motion of the plate: the plate is moving into, rather than out of, the slit.

In Fig. 2, the dependence of $\bar{u} = u/u_0$ and $\bar{v} = -v/\sqrt{x}/q$ on the self-similar variable η is compared with the analogous dependence corresponding to the formulation of the problem in [5] with the same suction intensity q.

For axisymmetric flow induced by the motion of a cylinder in the axial direction, the boundary-layer equations in the cylindrical coordinates r, z (the z axis coincides with the cylinder axis) are written in the form

$$u\frac{\partial u}{\partial z}+v\frac{\partial u}{\partial r}=\frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right),\quad \frac{\partial (ru)}{\partial z}+\frac{\partial (rv)}{\partial r}=0.$$

The non-self-similar accurate solution of these equations describing the flow close to a permeable cylinder of radius r_0 is written in parametric form

$$u=u_0\frac{t^2}{\alpha^2},\quad \alpha=\xi^2+t-1,\quad \xi=\frac{r}{r_0},$$

$$v=\frac{q}{\xi}\left[-2+\frac{6(t-1)}{\alpha}-\frac{3t(t-1)}{\alpha^2}\right],\quad q=\frac{2\nu}{r_0}, \tag{2}$$

$$z=z_*(t+\ln|t-1|+A),\quad z_*=\frac{u_0r_0^2}{12\nu}.$$

Here A is an arbitrary constant.

The distribution of the suction rate over the cylinder surface is obtained from Eq. (2) in the following parametric form

$$v_0=q\left(1-\frac{3}{t}\right),\quad z=z_*(t+\ln|t-1|+A). \tag{3}$$

Considering different regions of variation in t in Eq. (2) and different values of the constant A, different types of solutions may be obtained.

For t varying from 0 to 1 when $u_0 < 0$ and $A = 0$ ($z_* < 0$, so that z varies from 0 to ∞), a solution corresponding to motion of a cylinder at velocity $|u_0|$ in the direction opposite to the z axis is obtained. The distribution of the suction rate $v_0(z)$ calculated from Eq.

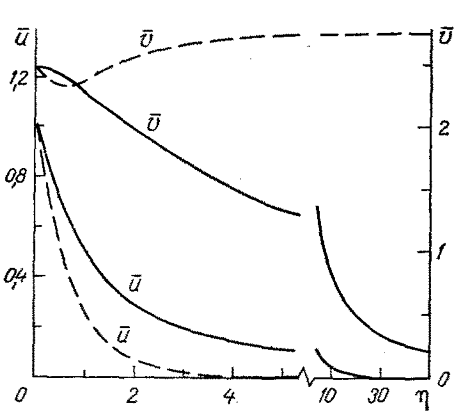

Fig. 2

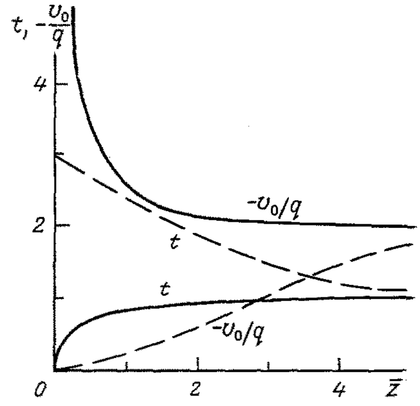

Fig. 3

Fig. 2. Comparison of the dependence of the dimensionless velocity components $\bar{u} = u/u_0$ and $\bar{v} = v/v_0$ on the self-similar variable η for the case of a plane surface with the analogous dependence from [5]: continuous curves) Eq. (1); dashed curves) [5].

Fig. 3. Dependence of the suction rate v_0 and the conditional boundary-layer thickness t on the dimensionless longitudinal coordinate $\bar{z} = z/|z_*|$ at a cylindrical surface for two types of solutions: continuous curves) Eqs. (2) and (3) with $0 \leq t \leq 1$, $A = 0$; dashed curves) Eqs. (2) and (3) with $1 \leq t \leq 3$, $A = -3 - \ln 2$.

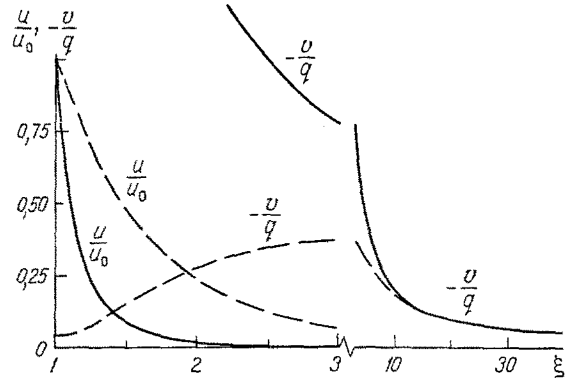

Fig. 4. Radial distribution ($\xi = r/r_0$) of dimensionless velocity components in the case of a cylindrical surface with $z/|z_*| = 0.2$. For two types of solution: continuous curves) Eq. (2) when $0 \leq t \leq 1$, $A = 0$; dashed curves) Eqs. (2) with $1 \leq t \leq 3$, $A = -3 - \ln 2$;

(3) with A = 0 is shown in Fig. 3. The variation in boundary-layer thickness along the cylinder is shown by the curve of t(z) in Fig. 3; the boundary-layer thickness δ is determined from the condition $u = ku_0$; hence, using Eq. (2), it follows that δ is proportional to t.

A solution of a different type is obtained from Eq. (2) with variation in t from 3 to 1 ($u_0 < 0$; $A = -3 - \ln 2$; $0 \leq z \leq \infty$), as shown by the dashed curves in Fig. 3. In contrast to the preceding solution, the suction rate falls here in the direction of cylinder motion, which leads to increase in boundary-layer thickness in this direction.

The radial distributions of the longitudinal and transverse velocity components corresponding to these two types of solution for the same value of the longitudinal coordinate $z/|z_*| = 0.2$ are shown in Fig. 4.

Parameter values $t \geq 3$ ($u_0 > 0$, $A = -3 - \ln 2$) may also be considered, corresponding to motion of the cylinder in the direction of the z axis in the presence of liquid injection through the surface.

NOTATION

x, y, Cartesian coordinates; r, z, cylindrical coordinates; r_0, cylinder radius; u, v, longitudinal and transverse (to the direction of surface motion) velocity components; u_0, velocity of surface motion; v_0, suction or injection rate through surface; q, parameter determining the suction or injection intensity; ν, kinematic viscosity; η, self-similar variable in Eq. (1); $\xi = r/r_0$, dimensionless radial coordinate; t, parameter in Eqs. (2) and (3) proportional to the boundary-layer thickness; δ, boundary-layer thickness; k, constant conditionally determining the boundary-layer thickness with respect to the degree of velocity drop at its boundary, $u = ku_0$ when $r = r_0 + \delta$; A, constant in Eqs. (2) and (3); z_*, combination of parameters with the dimensions of length, as defined in Eq. (2); α, auxiliary parameter defined in Eq. (2); $\overline{u} = u/u_0$, $\overline{v} = v/v_0$, $\overline{z} = z/|z_*|$, auxiliary parameters used in Figs. 2 and 3.

LITERATURE CITED

1. B. C. Sakiadis, AIChE J., 7, No. 1, 26-28 (1961).
2. B. C. Sakiadis, AIChE J., 7, No. 2, 221-225 (1961).
3. B. C. Sakiadis, AIChE J., 7, No. 3, 467-472 (1961).
4. E. A. Koldenhof, AIChE J., 9, No. 3, 411-418 (1963).
5. L. E. Erichson, L. T. Fan, and V. G. Fox, Ind. Eng. Ch. Fund., 5, No. 1, 19-25 (1966).

TURBULENT FLOW OF A FIBROUS SUSPENSION IN A PIPE

A. R. Kuznetsov, A. Ya. Ageev, and S. A. Kuznetsova

UDC 532.542.4

The Henky–Ilyushin equations are used to describe the steady turbulent flow of an incompressible viscoplastic fluid in a pipe. A fibrous suspension is examined as the fluid.

The class of viscoplastic fluids contains a large number of systems such as cement mortars, oil–sand mixtures, oils, coal suspensions, etc. [1]. Fibrous suspensions of cellulose and asbestos in turbulent flow regimes can also be regarded as viscoplastic fluids.

A large number of investigations have been made of the laminar flow viscoplastic fluids, while the turbulent flow of these fluids has received little attention. Thus, the authors of the monograph [1] examined different problems connected mainly with the laminar motion of viscoplastic media. Several studies [2-4] have examined the laws governing the motion of fibrous suspensions. Here, researchers have obtained an extensive amount of experimental data and have developed an empirical approach to the study of the turbulent flow of fibrous suspensions. The authors of [5, 6] examined the turbulent flow of sand suspensions with the use of equations for each phase and with allowance for interaction of the phases. This approach leads to very complicated relations which include several unknowns and require time-consuming numerical study. In connection with this, it is interesting to examine the study [7]. Here, continuum conservation equations for the phases of the suspension were obtained using Feynman integrals over trajectories.

The motion of viscoplastic media is described by the Henky–Il'yushin differential equations. These equations appear as follows in vectorial form for an incompressible fluid [1]:

Lenin Komsomol Institute of Wood Technology, Sverdlovsk.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 58, No. 2, pp. 223-229, February, 1990. Original article submitted October 6, 1988.